Sonoluminescence first discovered in 1933 [1], is a remarkable phenomenon whereby sound is converted into light. The process involving a small gas bubble trapped in a degassed liquid which is then acoustically driven [single bubble sonoluminescence (SBSL)] has been recently studied [2]. Since the sound wavelength is much larger than the bubble radius $R(t) < 50 \mu\text{m}$ (see Fig. 1), the bubble feels a uniform pressure which varies with time. Undergoing complicated nonlinear oscillations, the bubble crosses a threshold of the driving pressure of about 1.15 atm, when the acoustic energy is focused into a small region and short (<50 ps), intense pulses of light are emitted.

Recent interest in sonoluminescence has been partly motivated by possible technological applications in biophysics, sonochemistry, and nuclear reactions. SBSL experiments are also sensitive to ambient temperature, composition, and driving frequency [2,3]. Thermal blackbody and/or Bremsstrahlung radiation (BR) [2,3], Casimir effects [4], and collision induced emission [5] have all been suggested as mechanisms for luminescence. Spectral fits to blackbody or BR emission are inconclusive but suggest temperatures of at least 5000 K [2,3].

Theoretical attention has focused on bubble dynamics, usually employing variations of the Rayleigh-Plesset equation to model the bubble motion and stability [2,6–8]. Furthermore, studies imply that, as the gas bubble shrinks, it launches an inwardly propagating shock wave [7]. The shock collapses to the center, rebounds, and can hit the liquid/gas bubble wall from which it was launched. Radiation is thought to be emitted immediately after the shock rebounds from the center, where extremely high temperatures and ionization are predicted [7]. Numerical simulations suggest temperatures of $10^8$ K [7], much higher than those implied by experiments. However, the emission spectra in SBSL are so featureless that distinguishing among radiation mechanisms is difficult. Despite attention on the effects of bubble dynamics [2,8], diffusion [9], and material composition on SBSL [10], the underlying physical mechanisms remain an unsorted mixture of nonlinear hydrodynamics, shock physics, and chemical reaction kinetics. One apparent criterion for SBSL is the stability of spherical bubble structure $R(t)$. The transient symmetric stability of the converging shock is also thought to be crucial for SBSL. In this Letter, we suggest using an external magnetic field to break the spherical stability, disrupt SBSL, and/or change the emitted radiation pattern.

Effects of $\vec{B}_{\text{ext}}$ on bubble structure.—We assume that the bubble gas is ionized at some point in the acoustic cycle. Adiabatic compression and heating of the gas bubble $R(t)$ is probably sufficient to partially ionize the gas. The sensitivity of Saha’s equation [11] to temperature $T$ ($10^{-8}–0.73$ ionization fraction $q(T)$ for $T = 5000–10^8$ K, and $N_2$ ionization potential) is important in determining the conductivity ($\sigma_p$) and plasma frequency ($\omega_p$). First, consider the possibility that, during the adiabatic collapse, a spherical region of plasma has a high $\sigma_p$ such that the magnetic Reynolds number $R_M = \sigma_p v R^2 \gg 1$ (where $v$, $R$ are typical velocities and the radius of the ionized region). The field is then “frozen in” and adds magnetic stiffness perpendicular to $\vec{B}_{\text{ext}}$; the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The sonoluminescing bubble in an external magnetic field $\vec{B}_{\text{ext}}$. The liquid/gas interface and the shock front are denoted by $R(t)$ and $r(t)$, respectively. The external sound field supplies the pressure $P_e$.}
\end{figure}
ionized region collapses asymmetrically, with the poles compressing more than the equator. If \( R_M \gg 1 \) is not achieved by initial adiabatic compression, shocks launched by the liquid/gas interface may further compress, heat, and ionize the plasma, increasing \( \sigma_y \) and \( R_M \).

Shocks form when particle velocities exceed \( \dot{B} \)-dependent characteristic group velocities. For \( R_M \gg 1 \), the magnetoacoustic and Alfvén magnetohydrodynamics (MHD) waves have group velocities \( 2 \omega_{\text{in}} = (c_s^2 + \omega_1^2) \pm [c_s^2 + \omega_1^2 - 4c_s^2 \sin^2(\theta)]^{1/2} \) and \( v = \omega_1 \cos \theta \) [12], where \( c_s = B^2/4\pi \rho \) and \( c_\parallel \) is the hydrodynamic sound speed \( (B \) is the magnetic field in the region of interest, and \( \rho \) is the total plasma density). The angular dependence of these velocities will break the spherical symmetry of a MHD shock.

\[
\rho v \left( \hat{n} \cdot \nabla v_i + \frac{1}{2} n_i \nabla \cdot v \right) = \left[ \frac{(\hat{B} \cdot \hat{n})B_i}{4\pi} - pv_i(\hat{v} \cdot \hat{n}) - P_M n_i \right] \\
\eta \hat{n} \cdot (\nabla \times \hat{B}) \times \hat{B} = \hat{n} \cdot \left[ \hat{v} \left( u + \frac{1}{2} \rho v^2 + P_M \right) - \frac{(\hat{B} \cdot \hat{v})\hat{B}}{4\pi} \right] \\
4\eta \hat{n} \times \nabla \times \hat{B} = [\hat{n} \times (\hat{v} \times \hat{B})],
\]

where \( \hat{n} \) is the radial director, \( [S]^x_{\theta} = S(x) - S(0) \), and \( u = P/(\gamma - 1)\rho \) is the internal energy of the gas \( [P, v, \gamma = C_p/C_v, \text{and } \eta = c_s^2/16\pi \sigma_y \) are the pressure, shear viscosity, adiabatic index, and plasma resistivity, respectively]. The total pressure \( P_M = P + B_0^2/8\pi \) (the \( E^2 \) contribution is smaller by at least \( (\nu/c)^2/R_M \), which we assume small). The limits straddling the shock are \( (0, 0) \), far outside the shock where all quantities are uniform, and \( x, \) a normal distance downstream. We neglect the contribution of any radiation burst to the electromagnetic stress tensor by considering the shock conditions just before or after the burst. Compression ratios and pressure jumps depend on \( \theta \), the angle between the \( \hat{B}_0 \) and the shock normal. We thus expect a nonuniform MHD shock forming first at the poles (\( \theta = 0 \)), where \( \hat{B}_0 \) offers no additional stiffness.

In the high conductivity \( (R_M \rightarrow \infty) \) limit, all quantities vary over a thin shock front such that the fields at \( x = 0^- \) are uniform, and gradient terms vanish away from the sharp transition region. Upon solving the resulting equations (Rankine-Hugoniot), we obtain

\[
\frac{2\gamma}{\gamma - 1} \left( \frac{\Delta}{s} - 1 \right) + \gamma M_0^2 \left( \frac{1}{s} - 1 \right) = 4\beta^{-1}(1 - \alpha) \sin^2\theta - \gamma M_0^2 \tan^2\theta \left( 1 - \frac{\alpha}{s} \right)^2.
\]

\[
\Delta - 1 = (\alpha^2 - 1) \sin^2\theta,
\]

where \( \alpha = (\beta \gamma M_0^2 - 2 \cos^2\theta)/(\beta \gamma M_0^2/s - 2 \cos^2\theta) \), \( \beta = 8\pi P_0/B_0^2 \) is the ratio of the particle to magnetic pressure, and \( M_0 = u_0/c_s \) is the Mach number. The roots of Eqs. (2) \( s \) and \( \Delta \) (>1), determine the thermodynamically allowed shock states and are plotted in Fig. 2 for \( \gamma = 5/3 \) at various \( M_0 \) and \( \beta \). Except for low \( \beta \) and \( M_0 \), the compression and density ratios, \( \Delta \) and \( s \), incur their greatest jumps at the poles. Numerically predicted pressures are on the order \( 10^{10} \) dyne/cm\(^2\) [7], thus, fields of \( B_0 \sim 10 \) T are required for \( \beta = O(1) \). The angle \( \theta \) is roughly the angle of \( \hat{B}_{\text{ext}} \) in Fig. 1. Deformations of the ionized region before shock formation will stretch the \( x \) axes in Fig. 2 depending on \( \beta \). Figure 2, however, accurately shows the difference in jumps between

![FIG. 2. The pressure and density ratios for \( R_M \gg 1 \). (a) \( \Delta \) as a function of \( \theta \) for \( M_0 = 1.5 \) (lower three thin curves), and \( M_0 = 3 \) (upper three thick curves). Solid, dotted, and dashed curves correspond to \( \beta = 10, 1, 0.1 \), respectively. (b) Density ratio \( s \) for similar values of \( M_0 \) and \( \beta \).](image-url)
$\theta = 0, \pi/2$. For large $\beta$, the shock will be nearly spherical, but magnetic field compression near the equator would tend to decrease $\theta$ throughout the hemisphere. For small $\beta$, the shock surface will be oblate owing to the lateral magnetic stiffness, and, again, smaller $\theta$ will approximate most of the surface. In fact, the shock solution dissipates as a switch-off shock (Fig. 2) for low enough $\beta$.

In the $R_M \to 0$ limit the flow and magnetic field are decoupled, and the plasma can slide freely through the field. Discontinuities in $s$, $\Delta$, and $\vec{v}$ approach those of a hydrodynamic shock, and the effect of the external $\vec{B}$ field for $R_M \ll 1$ will be small. Though Eq. (2) is always correct, for small $R_M$, the $\vec{B}$-field dissipation length $L$ becomes much larger than the viscous dissipation scale $\delta$—the scale over which pressure and velocity vary. Instead of solving the nonlinear differential equations (1) [13], we estimate the variation of $B$ across $\delta = \pi/2$, where the largest effect occurs. Most of the variation in $\vec{v}$, $\rho$, and $P$ will occur over $\delta$; far enough away from the shock, differences in the uniform states (0 and 1) are given by (2). Using $dB/ds = (B_0 - B_1)/L$, and evaluating quantities in Eqs. (1) across $\delta$ to order $R_M/\beta$,

$$s = \tilde{s} - 2 \frac{[M^2_0(\gamma - 1) - \gamma]^2 + M^2_0 \gamma(\gamma - 1)}{[M^2_0(\gamma - 1) + \gamma]^2} \left( \frac{R_M}{\beta} \right) f(\theta)$$

$$\Delta = \tilde{\Delta} - \frac{2 \gamma M^2_0}{\gamma - 1} \left( \frac{M^2_0 \gamma - 3 \gamma + 1}{\gamma - 1} \right) \left( \frac{R_M}{\beta} \right) g(\theta),$$

where $\tilde{s} = (\gamma + 1)M^2_0/[\gamma + M^2_0(\gamma - 1)]$ and $\tilde{\Delta} = (4M^2_0 - \gamma + 1)/(\gamma + 1)$ are the density and compression ratios of a pure hydrodynamic shock. The angular dependences obey $f(0) = g(0) = 0$ and $f(\pm \pi/2) = g(\pm \pi/2) = 1$. As expected, the shock anisotropy decreases as $R_M/\beta$.

Effects of $\vec{B}$ on radiation.—Even when $R_M$ is too small for $B_{\text{ext}}$ to affect the shock, $B_{\text{ext}}$ can affect SBSL radiation. Although numerical studies [7] and experimental fits to blackbody and BR [2,3] imply vastly different peak temperatures ($10^8$ K vs $10^4$ K), thermal velocities $< 0.1c$ in both interpretations. Hence, we need only consider the nonrelativistic conditions for observing anisotropic cyclotron radiation.

We outline the possible radiation phenomena by considering a singly ionized species in a neutral plasma, justified because the Debye length is small ($< 1$ nm). If shocks do not appreciably affect the magnetic field, $B_0 = B_{\text{ext}} \tilde{z}$ within the plasma, and charge trajectories obey $\vec{v}_e = (e/m_e)\vec{v}_e \times B_0$, where $\vec{v}_e$ is the electron velocity. The dipole approximation for cyclotron emission power per unit solid angle of a collection of uncorrelated electrons is given by [11]

$$dL_e/d\Omega = (n_e e^4/4\pi m_e^2 c^5)v^2_e B^2_0 (1 - \sin^2 \theta \sin^2 \varphi),$$

where $n_e$ is the free electron density, $\hat{n}$ points to the observer, $\theta$ is the angle between $\vec{B}_{\text{ext}}$ and the line of sight, and $\varphi$ is the azimuthal angle to be averaged over. For $\omega_B^2 \ll 50$ ps, requires that $B_0 > 1 - 10$ T.

The integrated power as a function of $\theta$ is an average over $\varphi$ with

$$f(v_e)d\Omega = \exp[-v^2_e/2]v^4_e d\varphi,$$

where $v_B = (2k_B T/m_e)^{1/2}$. We also impose a cutoff on the velocity, $v_\perp < \ell \omega_B$, where $\ell = \min(A, R^*)$, and $\lambda$ is the mean free path for electron collisions. This ensures that the charge can coherently twist around the guiding center sufficiently to radiate near $\omega_B$. The cyclotron power is roughly

$$L_e(\theta) = \frac{n_e e^2}{c^5} \left( \frac{e^2 B_0 v_B}{5m_e} \right)^2 \left[ 1 - (\Lambda + 1)e^{-\Lambda} \right] \times \text{Erf} \left( \sqrt{\Lambda} (1 + \cos^2 \theta) \right),$$

where $A = (\ell \omega_B)^2 m_e/(2k_B T)$. When $\ell = \lambda < R^*$, $\Lambda \propto T^3$; but when $\ell = R^* < \lambda$, $\Lambda \propto 1/T$. Thus the range of $v_\perp, v_z$ maximally contributing to $L_e$ is where $\lambda \approx R^*$. When $\Lambda \gg 1$, $L_e(\theta) = n_e (6\pi^2 B_0^2 /16\pi^2 m_e^2 c^5) (1 + \cos^2 \theta)$, the standard expression [11,12]. When $\Lambda \ll 1$,

$$L_e(\theta) = \frac{n_e e^2}{c^5} \left( \frac{e^2 B_0 v_B}{5m_e} \right)^2 \left[ 1 + \cos^2 \theta \right] + O(\Lambda^3/2).$$

To determine how easily anisotropic emission may be observed, we compare the cyclotron intensities with possible isotropic BR emissions. Near $\omega_B$, the isotropic BR power in the dipole approximation is

$$L_{BR} = \int \frac{d\omega}{\omega - 1/2} \frac{dL_{BR}}{d\omega} = \frac{6n_e^2 e^6}{c^3 m_e^2 v_B^2} g_{ff} \Gamma,$$

where $b_{\min} \sim 4e^2/\pi m_e v^2_B$ is a minimum impact parameter, $g_{ff} = \ell n(v_B/\omega_B b_{\min})$ is the Gaunt factor, and $\Gamma$ is the cyclotron line width or detector bandwidth. Since detector bandwidths can be small, we use $\Gamma = \omega_{col} \sim v_B/\Lambda$ from collisional broadening, which dominates Doppler broadening for $T < 10^8$ K. The anisotropies $\Delta L_e = L_e(0) - L_e(\pi/2)$, compared to $L_{BR}$, are $\Delta L_e/\Lambda \gg 1/L_{BR} \sim 3.3 \times 10^{-3}(\omega_B/\Gamma) \times (\omega_B^2/m_e^2 v^2_B / g_{ff} e^4 n_e)$ and $\Delta L_e (\Lambda \ll 1)/L_{BR} \sim (\omega_B/\Gamma) \times (\omega_B^2 e^2 m_e^2 / 12 g_{ff} k_B T n_e e^4).$ For $e-e$ collisions, $\lambda = 4k_B T^2 / n_e e^4$. Here, we assume $e$-neutral scatterings are unimportant and $\Gamma < \omega_B$. Note the strong $\omega_B$ and $\ell \omega_B$ dependence of $\Delta L_e/L_{BR}$.

Discussion.—We have shown how a magnetic field $B_{\text{ext}}$ can break the symmetry of a collapsing ionized region or a propagating shock (Fig. 2) in two limits of $R_M$. Using Spitzer’s formula to estimate $\sigma_\gamma$ at $10^7$ K typical velocity and length scales from shock simulations [7] imply $R_M \sim 10^{-3}$. However, Guderly’s solution [14] for a spherical hydrodynamic shock gives $r(t) \sim r^0$, where $t_0$ is the time at implosion. Here, $v(t) \sim r^{-1}$, and $R_M \approx \sigma_p r^{2n-1}$. For noninteracting gases, $n > 1/2$, implying $R_M \to 0$ as the shock converges ($t \to 0$) and rebounds. Using gases having large $\gamma = C_p/C_v$ and
$n < 1/2$ when compressed and heated, if they exist, gives $R_M \rightarrow \infty$, where shocks will be strongly perturbed (Fig. 2).

For large or small $R_M$, anisotropies develop during the evolution of a shock. These anisotropies are greater for smaller $\beta$ (larger $B_0$), but diminish for small $R_M/\beta$. If SBSL requires a spherically converging shock, $B_{ext}$ may further destabilize the shock, reduce shock heating, and destroy luminescence. Comparison of experiments with numerical results can then produce an estimate of $R_M$. Furthermore, the $\theta$-dependent pressure exerted on the liquid/gas interface by a recolliding shock may destabilize the nonlinear bubble oscillations. Thus, an external magnetic field can alter the region of bubble stability and the threshold of SBSL. Differences between multiple bubble sonoluminescence (MBSL) and SBSL can also be probed; if only SBSL depends strongly on symmetric bubble oscillations, an external field would affect SBSL, not MBSL.

The radiation may also have an anisotropic cyclotron component given by Eq. (5). Since the luminescing spectrum is interrupted by the surrounding water for wavelengths $< 220$ nm, we suggest using $B_{ext}$ to study emission at longer IR and microwave wavelengths, where peaks near $\omega_B$ and its higher harmonics may be observed. One may be able to discern anisotropy at these wavelengths by comparing the low frequency spectra with and without $B_{ext}$ and estimating the cyclotron contribution by measuring $\Delta L_c/L_{BR}$. With our conjecture for $\lambda$, $\Delta L_c/L_{BR}$ implies that low $\eta_c(T)$, implying low $T$ and large $\omega_B$, enhances anisotropy and decreases $\omega_p$. However, with small $q(T)$, $e$-neutral scattering may become important, decreases $\lambda$ and broadens the anisotropy.

Guided by the expressions for radiated power in a magnetic field, experiments can place bounds on parameters such as $T$, $\eta_c(T)$, and $R^*$. For example, one can probe the conditions predicted by hydrodynamical simulations [7]. For $T \sim 10^7$ K and $R^* \sim 0.1 \mu m$, our model for $\lambda$ gives $\lambda \gg \ell_c \sim R^*$. Most of the gas [$\eta_c(T) = \rho \sim 200$ kg/m$^3$] is ionized and, except for extremely large $B_{ext}$, $\omega_p \gg \omega_B$. However, the plasma remains optically thin over lengths less than $\ell_d = 2\pi e/\omega_p \sim 0.2 \mu m$. For $B_{ext} = 100$ T, a small ($\lambda \ll 1$) but significant tail of the distribution $f(\nu_e)$ will support cyclotron radii $\nu_e/\omega_B < R^*$ and contribute to anisotropy when the radiating region $R^* \sim \ell_d$. If $\ell_c \sim R^* \sim 0.1 \mu m$ and BR is a competing process, the above parameters give $\omega_B/\Gamma \sim 200$ and $\Delta L_c/L_{BR} \sim 1/50$. For smaller $\ell_c$ or $B_0$, the anisotropy is strongly diminished; if $\ell_c = 0.01 \mu m$, $\Delta L_c/L_{BR} \sim 2 \times 10^{-7}$. Given the stability of SBSL, photon counting statistics may be a feasible way to detect small anisotropies. If $L_c(\theta)$ is detected at low $T$, collisions with neutrals are less important than expected. Furthermore, if the plasma is optically thick at $\omega_B$ (if $\omega_R > \omega_B$ and $\ell_d \ll R^*$), blackbody radiation, $L_{BB} = \omega_B^2 \Gamma k_B T/c^2 R^*$, would dominate.

We have not considered nonthermal (nonequilibrium) radiative processes. The true velocity distribution $f(\nu_e)$ may give quantitatively different anisotropies. Finally, note that experiments hitherto have time integrated the radiation. A time-dependent spectral analysis of asymmetric radiation near $\omega_B$ can probe the temperature and charge density of the heat-up and cool-down phases of the oscillation, in addition to the 50 ps burst.

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